On the effective indenter shape used in the analysis of nanoindentation unloading curves

GUANGHUI FU*
Lam Research Corporation, 4540 Cushing Parkway, Fremont, CA 94538, USA
E-mail: guanghui.fu@lamrc.com

LING CAO
University of California at San Francisco, 1472 5th Avenue 4-2, San Francisco 94143, USA

Nanoindentation experiments have become a commonly used technique to investigate mechanical properties of thin films and small volumes of materials. Effective indenter shape concept was introduced by Pharr and Bolshakov [1] to explain the nanoindentation unloading curves. They assume a certain pressure distribution and use linear elasticity theory to obtain the deformed surface profile. The effective indenter shape is derived from the deformed surface profile and represented by a power-law formula. Nanoindentation unloading curves are explained through Sneddon's solution [2] corresponding to this formula. However, their interpretation of the relationship between the pressure distribution and the effective indenter shape is not correct. Similar to Pharr and Bolshakov [1], we limit our discussions to the linear theory of elasticity and only uniform pressure distribution is considered.

We consider the deformation of a flat elastic half-space ($z \geq 0$) under a certain pressure distribution, $p(r)$, over a circular surface region ($0 \leq r \leq a$ and $z = 0$). The problem is considered in the linear theory of elasticity and the half-space is assumed to be isotropic and homogeneous. The following equations give the relevant displacement and stresses for the half-space. The vertical component of the displacement is denoted by u_z, and the stress components have two subscripts corresponding to the appropriate coordinates. E and ν are Young's modulus and Poisson's ratio of the half-space.

As Fig. 1 shows, the boundary conditions for the half-space at $z = 0$ are

$$\tau_{zr} = \tau_{z\theta} = 0, \quad (0 \leq r < \infty) \tag{1}$$

$$\sigma_{zz} = 0, \quad (r > a) \tag{2}$$

$$\sigma_{zz} = p(r), \quad (0 \leq r \leq a) \tag{3}$$

The deformed surface profile is given in the linear theory of elasticity as [3]

$$u_z(r) = \frac{4(1-\nu^2)}{\pi E}\left[\int_0^r \frac{s}{r} K\left(\frac{s}{r}\right) p(s)\,\mathrm{d}s + \int_r^a K\left(\frac{r}{s}\right) p(s)\,\mathrm{d}s\right], \quad (0 \leq r < a) \tag{4}$$

where

$$K(k) = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1 - k^2 \sin^2\theta}}\mathrm{d}\theta, \quad (0 \leq k < 1)$$

(complete elliptic integral of the first kind).

For a uniform pressure distribution, Equation 3 becomes $\sigma_{zz} = q$ and Equation 4 is simplified as

$$u_z(r) = \frac{4(1-\nu^2)qa}{\pi E} E\left(\frac{r}{a}\right), \quad (0 \leq r < a) \tag{5}$$

where

$$E(k) = \int_0^{\frac{\pi}{2}} \sqrt{1 - k^2 \sin^2\theta}\, d\theta, \quad (0 \leq k \leq 1)$$

(complete elliptic integral of the second kind).

One of the assumptions in the linear theory of elasticity is that the displacement-gradient components are small compared to unity, i.e., the theory is valid only for small strains and small rotations. In the following discussion, we will check whether or not Equation 5 satisfies the small rotation condition.

From Equation 5, we have the rotation of the surface normal vector (see Fig. 2) within the loading area as

$$\theta(r) = \arctan\left(\left|\frac{\mathrm{d}u_z(r)}{\mathrm{d}r}\right|\right) = \arctan\left(\left|\frac{4(1-\nu^2)q}{\pi E} \cdot \frac{E(r/a) - K(r/a)}{r/a}\right|\right), \quad (0 \leq r < a) \tag{6}$$

When $r \to a^-$, $K(r/a) \to \infty$ and $\theta(r) \to 90^\circ$, i.e., near the edge of the loading area, the surface normal rotation is not small. This violates the small rotation assumption in the linear elasticity theory. From this, we conclude that Equation 5 is not valid near the edge of the loading area.

Further investigation shows that whether or not Equation 5 is valid in other areas of the loading region depends on the ratio, $(1-\nu^2)q/E$. We consider three materials used by Pharr and Bolshakov [1] (Table I).

*Author to whom all correspondence should be addressed.

TABLE I Material properties used by Pharr and Bolshakov [1]

Material	E (GPa)	H (GPa)	ν	$(1-\nu^2)H/E$
Soda-lime glass	70.00	5.90	0.230	0.0798
Sapphire	403.00	26.90	0.234	0.0631
Fused silica	72.00	8.40	0.170	0.1133

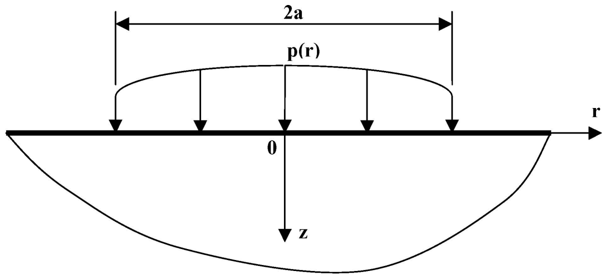

Figure 1 Axisymmetric normal loading on a circular surface area of an elastic half space.

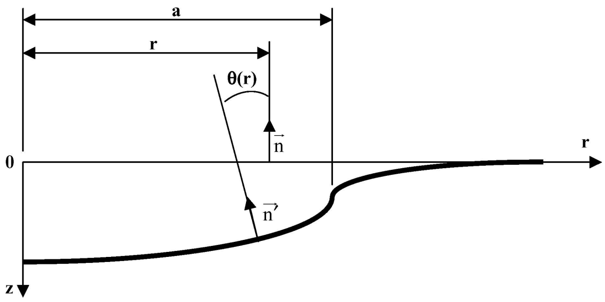

Figure 2 The rotation of the surface normal vector.

Following the same procedure as Pharr and Bolshakov and replacing the uniform pressure (q) by the material hardness (H), we have the corresponding rotation of the surface normal for each material shown in Fig. 3.

From Fig. 3, Equation 5 is clearly not valid for soda-lime glass, sapphire and fused silica for most of the loading areas. Thus, the ratio, $(1-\nu^2)q/E$, has to be small enough in order to satisfy the small rotation condition.

Summarizing the discussions, we have that Equation 5, a result from the linear elasticity theory, is not valid near the edge of the loading area. To make it valid in other regions of the loading area, the ratio $(1-\nu^2)q/E$ has to be small enough.

According to Pharr and Bolshakov [1], the effective indenter shape corresponding to the uniform

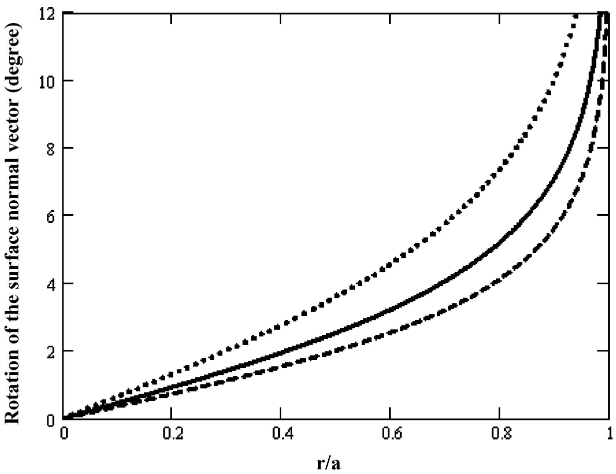

Figure 3 $\theta(r)$ vs. r/a for different materials. Solid line is for soda-lime glass, dashed line is for sapphire and dotted line is for fused silica.

loading of a circular surface area of an elastic half-space is

$$f(r) = \frac{4(1-\nu^2)qa}{\pi E}\left[\frac{\pi}{2} - E\left(\frac{r}{a}\right)\right], \quad (0 \le r \le a) \tag{7}$$

Equation 7 corresponds to Equation 5. In the linear theory of elasticity, the pressure at the edge of the contact area for a smooth indenter is either zero or infinite [4]. The effective indenter described by Equation 7 is a smooth indenter. At the edge of its contact area, the pressure cannot be kept at a non-zero finite value. Thus, there is no corresponding effective indenter for Equation 5. Even if Equation 5 is valid in the linear theory of elasticity throughout the loading region, the corresponding effective indenter, which gives a uniform pressure distribution, will not exist.

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